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Description of the model framework for immigration

The following is an economic model that describes supply and demand in the labor market. The model includes four population groups, in accordance with Card's (2009)[1] analysis: 1) high-education immigrant population, 2) high-education native-born population, 3) low-education immigrant population, and 4) low-education native-born population. The superscript indicates the type of parameter or variable (h=high education, l=low education, i=immigrant, n=native-born). Thus, for example, L^{hi} is the labor input of the highly educated immigrant population in the labor market.

The model is predicated on a production function that describes how production is created by combining inputs. The production function used is in the Cobb-Douglas function:

$$Y_t = AK_t^{\alpha} L_t^{\beta}, \tag{1}$$

where Y is total production, K is capital and L is labor input. Assumed are constant returns to scale, i.e., $\alpha + \beta = 1$, and also assumed that there is plenty of available physical capital and that it adapts to changes in labor input.

The production function is a collection of nested CES functions:

$$L_t = \left(\theta^h \, {L_t^h}^{\frac{\delta-1}{\delta}} + \left(1-\theta^h\right) \, {L_t^l}^{\frac{\delta-1}{\delta}}\right)^{\frac{\delta}{\delta-1}}, \tag{2}$$

$$L_t^h = \left(\theta^{hi} \, {L_t^{hi}}^{\frac{\sigma-1}{\sigma}} + \left(1-\theta^{hi}\right) \, {L_t^{hn}}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{3}$$

$$L_t^l = \left(\theta^{li} \, {L_t^{li}}^{\frac{\sigma-1}{\sigma}} + \left(1-\theta^{li}\right) \, {L_t^{ln}}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{4}$$

In the equations, θ is the productivity weight of each group and σ and δ are elasticities of substitution that describe the substitutability of the two factors of production. The parameter δ is the elasticity of substitution of those with high and low education, and σ measures the elasticity of substitution of immigrants and the native-born population. Marginal productivity determines the wage level (w) and, conversely, the demand for labor – in other words, $\frac{\partial Y_t}{\partial L_t^{ij}} = w_t^{ij}, i \in \{h, l\}$ and $j \in \{i, n\}$:

$$w_t^{hn} = \left(1-\theta_t^{hi}\right) \theta_t^h \, \beta \, {K_t}^{\alpha} \, {L_t}^{\beta-1+\frac{1}{\delta}} \, {L_t^h}^{\frac{1}{\sigma}-\frac{1}{\delta}} \, {L_t^{hn}}^{\frac{(-1)}{\sigma}} \tag{5}$$

$$w_t^{hi} = \theta_t^{hi} \, \theta_t^h \, \beta \, {K_t}^{\alpha} \, {L_t}^{\beta-1+\frac{1}{\delta}} \, {L_t^h}^{\frac{1}{\sigma}-\frac{1}{\delta}} \, {L_t^{hi}}^{\frac{(-1)}{\sigma}} \tag{6}$$

[1] Card (2009) *Immigration and Inequality*, American Economic Review: Papers & Proceedings 2009, 99:2, 1-21.

$$w_t^{ln} = \left(1-\theta_t^{li}\right)\left(1-\theta_t^{h}\right)\beta\, {K_t}^{\alpha}\, {L_t}^{\beta-1+\frac{1}{\delta}}\, {L_t^{l}}^{\frac{1}{\sigma}-\frac{1}{\delta}}\, {L_t^{ln}}^{\frac{(-1)}{\sigma}} \tag{7}$$

$$w_t^{li} = \theta_t^{li}\left(1-\theta_t^{h}\right)\beta\, {K_t}^{\alpha}\, {L_t}^{\beta-1+\frac{1}{\delta}}\, {L_t^{l}}^{\frac{1}{\sigma}-\frac{1}{\delta}}\, {L_t^{li}}^{\frac{(-1)}{\sigma}} \tag{8}$$

The description of supply of labor is somewhat simpler than demand and is determined by the individual optimization problem, where an individual maximizes their utility from consumption and the disutility of working subject to budget constraint.

$$\max_{l_t^{ij}} u(c_t^{ij}, l_t^{ij}) = \frac{\left(c_t^{ij}\right)^{1-e}}{1-e} - \gamma^{ij}\frac{\left(l_t^{ij}\right)^{1+\phi}}{1+\phi} \tag{9}$$

$$\text{s.t.}$$

$$c_t^{ij} = w_t^{ij} l_t^{ij}, \tag{10}$$

where c refers to consumption, e is the parameter that defines risk aversion, ϕ is the inverse of the Frisch elasticity of labor supply and γ is the parameter that defines the ratio of the utility of consumption and the disadvantages of working. The result is a set of simple labor supply functions:

$$l_t^{hn} = \left(\frac{1}{\gamma_t^{hn}}\right)^{\frac{1}{\phi+e}} {w_t^{hn}}^{\frac{1-e}{\phi+e}} \tag{11}$$

$$l_t^{hi} = \left(\frac{1}{\gamma_t^{hi}}\right)^{\frac{1}{\phi+e}} {w_t^{hi}}^{\frac{1-e}{\phi+e}} \tag{12}$$

$$l_t^{ln} = \left(\frac{1}{\gamma_t^{ln}}\right)^{\frac{1}{\phi+e}} {w_t^{ln}}^{\frac{1-e}{\phi+e}} \tag{13}$$

$$l_t^{li} = \left(\frac{1}{\gamma_t^{li}}\right)^{\frac{1}{\phi+e}} {w_t^{li}}^{\frac{1-e}{\phi+e}} \tag{14}$$

The labor supply functions for the national economy are obtained by combining the labor supply of an individual (lowercase letter) and the size of the population (N^{ij}):

$$L_t^{hn} = l_t^{hn}\, N_t^{hn} \tag{15}$$

$$L_t^{hi} = l_t^{hi}\, N_t^{hi} \tag{16}$$

$$L_t^{ln} = l_t^{ln}\, N_t^{ln} \tag{17}$$

$$L_t^{li} = l_t^{li}\, N_t^{ln} \tag{18}$$

The labor market balances are found at the intersections of labor supply and demand.

Parameters and model calibration

Prior to analysis, the model must be calibrated. In practice, the rate of employment of different population groups is extracted from the data, and using this information, the unknown parameters of the labor supply function are set to the correct levels. In addition, the relative wages are extracted from the data and are used to determine population group-specific productivity parameters. Thus, in equilibrium, $l^{hn} = 0.8, l^{hi} = 0.7, l^{ln} = 0.66, l^{li} = 0.56, w^{hn}/w^{ln} = 1.35$ and $w^{hn}/w^{hi} = 1.2$. On the basis of this information, the values for the following parameters can be determined: $\theta^{hi}, \theta^{h}, \gamma^{hn}, \gamma^{hi}, \gamma^{ln}$ and γ^{li}.

In addition, the model contains a set of parameters whose values are extracted from existing literature: $\alpha = 1/3$, $\beta = 1 - \alpha$, $\delta = 2$, $\sigma = 20$, $\phi = 1$, $e = 0.5$ and $\frac{k}{y} = 3$.

The shares of the population (N^{ij}) are calibrated as follows. Total population size is normalized to 1,000. According to the employment statistics of Statistics Finland, approximately six percent of the working-age population are foreigners. According to Statistics Finland's statistics on the educational attainment of the population, about 26% of persons with a foreign background have completed an advanced degree, and about 33% of those with a Finnish background have done so.